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Experimental examination of facilities layout problems in logistics systems including objects with diverse sizes and shapes

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Abstract. This study demonstrates a simulation experiment results regarding a flexible approach to solving facilities layout problem on a regular grid. In this paper approach we model objects with different dimensions and various shapes. We have implemented our version of the simulated annealing algorithm (Kirkpatrick et al., 1983) and analyzed how the number of cycles (100, 200, 300), objects types (uniform, diverse) affect average goal function values, mean concentration degrees, and the number of disintegrated objects. The formal statistical analysis was conducted separately for two sizes of regular grids, i.e. 6×6 and 10×10. The outcomes generally showed a significant influence of the studied effects on the analyzed dependent variables.

Keywords: logistics · facilities layout problem · simulated annealing · flexible approach · simulation experiment

1 Introduction

Since the facilities layout (FL) problem is significant in various areas of operations research, it has been subject to numerous studies throughout the years. The problem may be formulated as finding such an optimal arrangement of objects (e.g. machines or plants) that transport costs are minimized. Sahni and Gonzalez in 1976 demonstrated that FL belongs to at least NP-complete class of problems, thus it is not possible to find the optimal solution in polynomial time. Extensive descriptions of available methods for solving FL problems along with their typology are available, for instance, in Kusiak and Hergau (1987) or Singh and Sharma (2006).

Many of the classical algorithms require some restrictions in modeling real situations. For instance it is very popular to use regular grids as models of space available for designing layouts. In such cases, possible objects locations are predefined and it is usually assumed that objects have square shapes with the same dimensions. These requirements were naturally considered as drawbacks and as early as in the 1970s Armour and Buffa (1963) proposed an algorithm which allowed for including objects with various sizes. In their method, objects could consist of many unitary building blocks corresponding to individual grid cells. Thanks to that it was also possible to define objects with diverse shapes. The problem of finding optimal shapes was subsequently analyzed by Kim and Kim (1995). More flexible approaches to modeling objects with different sizes and shapes were also developed in a number of genetic algorithms. For example the concept of flexible bays (Tong, 1991; Tate and Smith 1995) or slicing tree methods (Tam, 1992; Liu and Sun, 2012).

In the present study we continue research in examining a flexible approach to objects dimensions and sizes located on a regular grid. Unlike Armour and Buffa (1963) or Bazaraa (1975) we propose an application of simulated annealing concept to specifically defined relationships where only zero-one values are used for describing associations between objects. Additionally, in contrast to Bazaraa (1975) or CRAFT (Buffa et al., 1964) algorithms in this proposal we do not put any restrictions on objects' shapes. Each object consists of the appropriate number of unitary building blocks. One of these components represents always the input – output place while the rest items are linked with it. This relationship value amounts to one.

To test our approach we designed and conducted a simulation experiment which was focused on relatively simple FL problems on a regular grid. The methods used and all the obtained results are presented and thoroughly analyzed in the following sections.

2 Methods

2.1 Algorithm implementation

We implemented a Simulated Annealing algorithm following the general recommendations provided by Kirkpatrick et al. (1983) to produce reasonable solutions on a regular grid. Some other modifications of the Simulated Annealing algorithm may be found in Meller & Bozer (1996) or recently in Kulturel-Konak & Konak (2015). This study approach resembles the physical process of metal particles' movements during the annealing process. In our facility layout context, these movements correspond to exchanging objects' components places. The whole process evolves along with decreasing the temperature (T). There also exists a possibility of accepting worse solutions with a predefined probability. The number of objects' components pair changes in a specific temperature is proportional to the number of objects' item problem $k \times N$, where k amounted to 10 in this study. The temperature changes are calculated as follows: $Tj = r \times Tj\text{-}1$, with $r = 0.9$. The stop criterion determined by this relation $Tf = 0.9(i - 1)\times Ti$, where $i = 100$ is the number of steps.

2.2 Dependent measures

The classical goal function formula was used as a dependent measure in our simulation experiments:

$$\text{Euclidean based goal function (OF)} = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left(D_{p(i)p(j)} \cdot L_{ij} \right), \tag{1}$$

where $D_{p(i)p(j)}$ is the standard Euclidean distance obtained according to the following formula:

$$D_{p(i)p(j)} = \left(\sum_{k=1}^{N} \left| x_{p(i)k} - x_{p(j)k} \right|^2 \right)^{1/2}, \tag{2}$$

with k denoting the dimension – here equal two, *and* L_{ij} specifying the relationship strength between two objects.

2.3 Experimental design and procedure

In our simulation experiments we confined to three variates, that is (i) the number of objects' components arranged in a square regular grid (6×6 = 36 and 10×10 = 100), (ii) the type of objects, that is their complexity: either all included the same number of items (six 6-item objects for the 6×6 grid and twenty objects consisting of 5 items each for the 10×10 grid) or included a diverse number of components (for the 6×6 grid: 4 objects with 6 items and 4 objects with 3 components; for the 10×10 grid: 10 objects with three components and ten objects with 7 elements), and finally (iii) the number of cycles applied in the SA algorithm (100, 200, 200).

We employed a full experimental design, which produced 12 different simulation conditions: *Grid size* (2) × *Objects type* (2) × *No of SA Cycles* (3), and replicated the simulation 100 times for each of the 12 conditions. We were interested not only in minimizing goal function values but also in how the objects items behave in terms of their shape and possible objects' disintegrations. Therefore, we analyzed both the Euclidean based goal function as well as the average concentration degree of the examined objects and a number of objects that were divided into two or more parts. The concentration degree was computed by dividing the bigger by the smaller object dimension. In this way, the biggest possible concentration degree amounted to 1 when the objects' items formed a square whereas smaller values indicated how far a specific shape was from being a square.

The distance between all neighboring objects' items was set at one and we assumed that each link between objects' components amounted to one.

3 Results and discussion

3.1 Basic descriptive statistics

For all three dependent variables and all 12 experimental conditions described in a previous section, we computed the minimum, mean, and mean standard error values. These basic descriptive statistics are put together in Table 1.

It may be observed that the obtained minimal values for the Euclidean based goal function for the 6×6 grid actually do not depend on the number of cycles applied in a SA algorithm which is not the case for the 10×10 grid. On the other hand the average values seem to decrease significantly along with the increase of the number of cycles used. For the concentration degree there seems to be a clear pattern visible only for the larger grid: the mean concentration increased for the bigger number of cycles.

Table 1. Basic descriptive statistics for all experimental conditions and all three dependent variables analyzed in the current study.

Condition	**Euclidean based goal function**			**Average concentration degree**			**Number of disintegrated objects**		
	Min	**Mean**	***MSE**	**Min**	**Mean**	***MSE**	**Min**	**Mean**	***MSE**
06×06-U-100	40.7	44.7	0.18	0.62	0.77	0.0068	0	0	0
06×06-U-200	40.7	43.7	0.19	0.60	0.78	0.0076	0	0	0
06×06-U-300	40.7	43.4	0.16	0.62	0.79	0.0074	0	0.02	0.020
06×06-D-100	42.0	46.2	0.17	0.50	0.79	0.0097	0	0.01	0.010
06×06-D-200	41.3	45.4	0.18	0.50	0.78	0.010	0	0.01	0.010
06×06-D-300	41.3	45.2	0.16	0.50	0.78	0.011	0	0.02	0.014
10×10-U-100	146	164	0.73	0.59	0.72	0.0043	0	2.96	0.160
10×10-U-200	139	153	0.70	0.62	0.73	0.0043	0	0.74	0.086
10×10-U-300	133	147	0.71	0.65	0.74	0.0036	0	0.28	0.049
10×10-D-100	151	165	0.69	0.59	0.76	0.0050	0	3.58	0.157
10×10-D-200	135	153	0.66	0.65	0.78	0.0053	0	1.57	0.078
10×10-D-300	135	148	0.68	0.64	0.80	0.0056	0	1.19	0.058

* MSE – Mean Standard Error, U – Uniform and D – Diverse no of objects' components

The presented data also suggest that for the larger grid, the mean number of disintegrated objects was decidedly smaller when the bigger number of SA cycles was employed. Moreover, it is worth noting that it was possible to obtain at least one solution where none of the objects was parted for all examined experimental conditions.

3.2 Analysis of variance

The described observations in a previous section are formally verified by means of analyses of variance. Since there was a significant difference between goal function values for 6×6 and 10×10 grids, a series of two-way analyses of variance were applied separately to check if the examined factors are statistically.

Analysis of variance for the 6×6 arrays of objects' components. The obtained ANOVA results for the Euclidean based goal function are summarized in Table 2. They revealed that both examined factors, that is objects type and the number of cycles significantly influenced the mean goal function. The interaction between these effects was irrelevant. The mean goal function values for both factors are illustrated in Figures 1 and 2.

Table 2. Two-way ANOVA regarding the influence of objects' type (uniform versus diverse number of object's components) and the number of SA cycles (100, 200, 300) on the goal function for 6×6 arrays of objects.

Effect	SS	df	MS	F	p	η^2
Objects type (O)	408	1	408	137	$< 0.0001^*$	0.188
Cycles (C)	153	2	77	25.8	$< 0.0001^*$	0.0799
O × C	3.12	2	1.56	0.526	0.591	0.00177
Error	1764	594	2.97			

$^*p<0.05$; df–degrees of freedom; SS–sum of squares; MS–mean sum of squares; η^2–partial eta-squared

Figure 1 shows that mean goal functions were significantly smaller than for diverse objects. The results presented in Figure 2 formally support the outcomes of the descriptive statistics analysis.

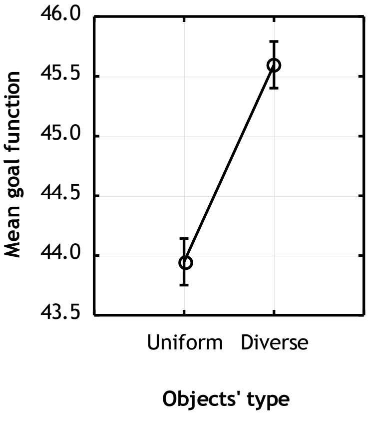

Fig. 1. The influence of objects' type (uniform versus diverse number of object's components) on the Euclidean based goal function for 6×6 arrays. $F(1, 594) = 137$, $p < 0.0001$. Vertical bars denote 0.95 confidence intervals

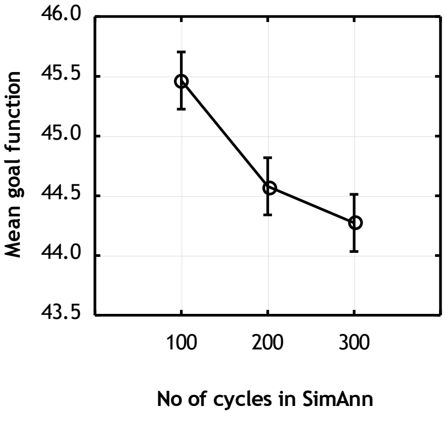

Fig. 2. The influence of the number of SimAnn cycles (100, 200, 300) on the Euclidean based goal function for 6×6 arrays. $F(2, 594) = 22.8$, $p < 0.0001$. Vertical bars denote 0.95 confidence intervals.

The average concentration degree of objects for 6×6 arrays occurred to be significantly influenced ($\alpha = 0.01$) neither by the examined factors nor by their interaction. A similar situation was observed for the number of disintegrated objects where the tested effects and their interaction were not statistically meaningful ($\alpha = 0.01$).

Analysis of variance for the 10×10 arrays of objects' components. The results of 10×10 array ANOVA are given in Table 3 and show that the mean goal function was significantly affected only by the number of SA cycles. The meaningful relation is presented in Figure 3.

Table 3. Two-way ANOVA regarding the influence of objects' type and the number of SA cycles on the goal function for 10×10 arrays.

Effect	SS	df	MS	F	p	η^2
Objects type (O)	64.3	1	64.3	1.33	0.249	0.00224
Cycles (C)	30523	2	15262	316	< 0.0001*	0.516
O × C	4.12	2	2.06	0.0426	0.958	0.000144
Error	28671	594	48.3			

*$p<0.05$; df–degrees of freedom; SS–sum of squares; MS–mean sum of squares; η^2–partial eta-squared

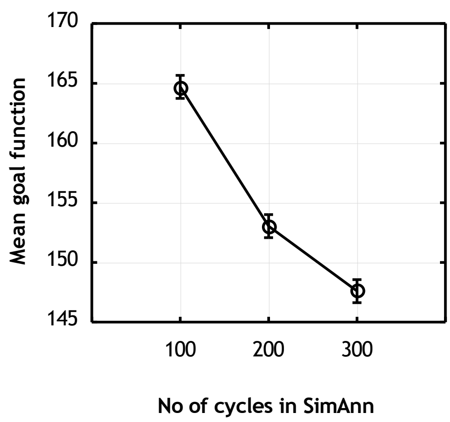

Fig. 3. The influence of the number of SA cycles on the goal function for 10×10 arrays. $F(2, 594) = 316$, $p < 0.0001$. Vertical bars denote 0.95 confidence intervals.

Another two-way ANOVA was applied to check whether examined factors affect the average concentration degree. The outcomes are demonstrated in Table 4 and Figures 4-6 and present a significant influence of both factors and their interaction.

Table 4. Two-way ANOVA regarding the influence of objects' type and the number of SA cycles on the average concentration degree for 10×10 arrays.

Effect	SS	df	MS	F	p	η^2
Objects type (O)	0.370	1	0.370	166	< 0.0001*	0.218
Cycles (C)	0.0807	2	0.0404	18.1	< 0.0001*	0.0574
O × C	0.0137	2	0.0069	3.07	0.0469*	0.0102
Error	1.33	594	0.00223			

*p<0.05; df–degrees of freedom; SS–sum of squares; MS–mean sum of squares; η^2–partial eta-squared

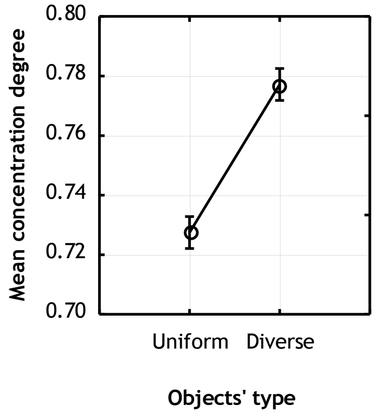

Fig. 4. The influence of objects' type on the mean concentration degree for 10×10 arrays. F(1, 594) = 166, p < 0.0001. Vertical bars denote 0.95 confidence intervals

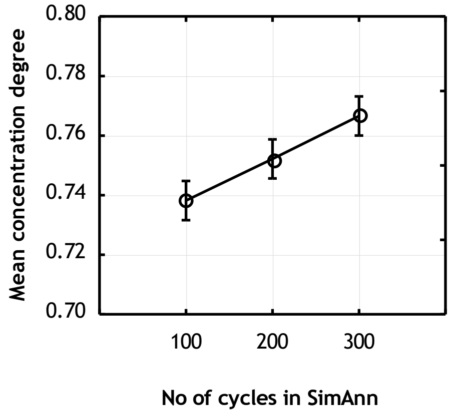

Fig. 5. The influence of the number of SA cycles on the mean concentration degree for 10×10 arrays. F(2, 594) = 18.1, p < 0.0001. Vertical bars denote 0.95 confidence intervals.

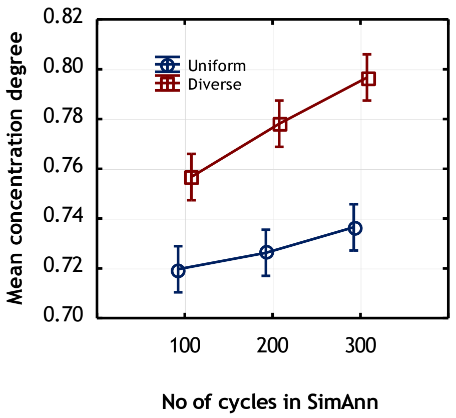

Fig. 6. The influence of the analyzed factors interaction on the mean concentration degree for 10×10 arrays. F(2, 594) = 3.07, p < 0.0469. Vertical bars denote 0.95 confidence intervals.

Finally, another ANOVA was employed to verify if the investigated effects significantly influenced the number of disintegrated objects. The results are provided in Table 5 and illustrated in Figures 7 and 8. The analysis revealed that both the type of objects as well as the number of SA cycles significantly differentiated the number of disintegrated objects.

Table 5. Two-way ANOVA regarding the influence of objects' type and the number of SA cycles on the number of disintegrated objects arranged in a 10×10 array.

Effect	SS	df	MS	F	p	η^2
Objects type (O)	92.8	1	92.8	80.0	< 0.0001	0.119
Cycles (C)	738	2	369	318	< 0.0001	0.517
O × C	2.24	2	1.12	0.966	0.381	0.00324
Error	690	594	1.16			

*p<0.05; df–degeers of freedom; SS–sum of squares; MS–mean sum of squares; η^2–partial eta-squared

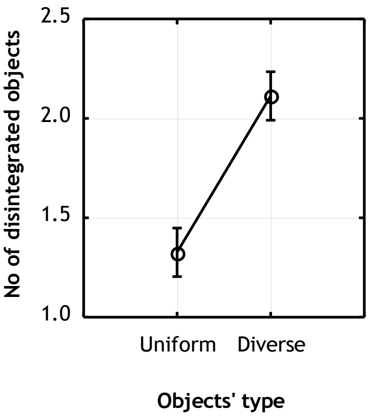

Fig. 7. The influence of objects' type on the number of disintegrated objects for 10×10 arrays. $F(1, 594) = 80$, $p < 0.0001$. Vertical bars denote 0.95 confidence intervals.

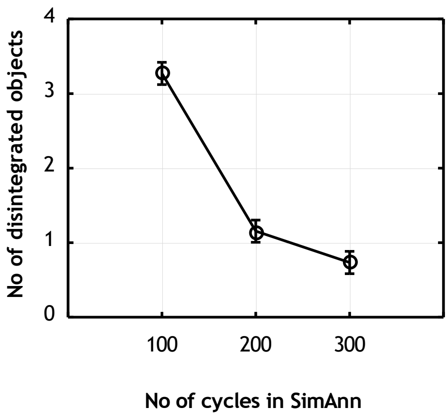

Fig. 8. The influence of the number of SA cycles on the number of disintegrated objects for 10×10 arrays. $F(2, 594) = 318$, $p < 0.0001$. Vertical bars denote 0.95 confidence intervals.

4 Discussion and conclusions

The liberalization of restrictions regarding objects' shapes is the crucial element of the presented approach. Every object is constructed from uniform, modular building blocks of the size 1×1. The relationships used in our experiment refer to the necessity of the given pair of objects to be located next to each other. The zero value means that such a requirement is not needed while one denotes that their proximity is essential.

Since links between whole objects and between objects' components are treated exactly the same, the total value of the goal function may be interpreted as an estimated joint cost of transportation between departments and within them, that is, between the input-output place and other objects' items representing single workstations inside departments.

The lack of other constraints may, however, result in disintegration of departments (objects) into smaller, not necessarily adjacent fragments. Still, the conducted simulation experiments showed that in each of the experimental conditions it was possible to obtain at least one solution where none of the objects was separated. It also seems that solutions with a small degree of objects' disintegration may be interesting for logistics systems designers. First, one may ponder whether the disintegration proposed by the algorithm could be a better solution and lead to lowering the transport costs despite dividing or even doubling some of the departments. Furthermore, some of the modern transportation systems (e.g. gantry robots) do not require a physical neighborhood of the cooperating workstations.

The results regarding the departments' concentration degree coefficients (measured by a ratio of the shorter side to the longer one) demonstrated that for the FL problems with binary relationships the increase of the number of cycles in the simulation annealing algorithm generates more condensed solutions. Additionally, these condensed designs provide more optimal results which suggests that objects' shapes obtained in the presented method may significantly influence the goal function value.

The promising findings presented in this paper point out potential directions of research and developments of the current study approach. It seems that the most interesting would be the examination of the algorithms' behavior in various types of transportation systems inside departments and investigations of diverse inter- and intradepartmental relationships and levels of transportation costs. In particular, the examination these parameters influence on the proposed departments/objects concentration and disintegration coefficients.

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References

1. Armour, G. C., & Buffa, E. S. (1963). A Heuristic Algorithm and Simulation Approach to Relative Location of Facilities. Management Science, 9(2), 294–309. doi:10.1287/mnsc.9.2.294
2. Bazaraa, M. S. (1975). Computerized Layout Design: A Branch and Bound Approach. *A I I E Transactions*, *7*(4), 432–438. doi: 10.1080/05695557508975028
3. Buffa, E. S., Armour, G. C., & Vollmann, T. E. (1964). Allocating Facilities with CRAFT. Harvard Business Review, 42(2), 136–158.
4. Kim, J.-Y., & Kim, Y.-D. (1995). Graph theoretic heuristics for unequal-sized facility layout problems. *Omega*, *23*(4), 391–401. doi:10.1016/0305-0483(95)00016-H
5. Kirkpatrick, S., Gelatt, C. D., & Vecchi, M. P. (1983). Optimization by Simulated Annealing. Science, 220(4598), 671–680. doi:10.1126/science.220.4598.671

6. Kulturel-Konak S., Konak A. (2015). A large-scale hybrid simulated annealing algorithm for cyclic facility layout problems. Engineering Optimization, 47(7), 963-978. doi: 10.1080/0305215X.2014.933825
7. Kusiak, A., & Heragu, S. S. (1987). The facility layout problem. European Journal of Operational Research, 29(3), 229–251. doi:10.1016/0377-2217(87)90238-4
8. Liu, X., & Sun, X. (2012). A multi-improved genetic algorithm for facility layout optimisation based on slicing tree. International Journal of Production Research, 50(18), 5173–5180. doi:10.1080/00207543.2011.654011
9. Meller R. D., Bozer Y. A. (1996). A new simulated annealing algorithm for the facility layout problem. International Journal of Production Research. 34(6), 1675–1692. doi: 10.1080/00207549608904990
10. Sahni, S., & Gonzalez, T. (1976). P-Complete Approximation Problems. J. ACM, 23(3), 555–565. doi:10.1145/321958.321975
11. Singh, S. P., & Sharma, R. R. K. (2006). A review of different approaches to the facility layout problems. The International Journal of Advanced Manufacturing Technology, 30(5-6), 425–433. doi:10.1007/s00170-005-0087-9
12. Tam, K. Y. (1992). A simulated annealing algorithm for allocating space to manufacturing cells. International Journal of Production Research, 3*0*(1), 63–87. doi:10.1080/00207549208942878
13. Tate, D. M., & Smith, A. E. (1995). Unequal-area facility layout by genetic search. *IIE Transactions*, *27*(4), 465–472. doi:10.1080/07408179508936763
14. Tong, X. (1991). SECOT: A Sequential Construction Technique for Facility Design. Unpublished Doctoral Dissertation, Department of Industrial Engineering, University of Pittsburgh.