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Simulated annealing based on linguistic patterns: experimental examination of properties for various types of logistic problems

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Abstract. The paper presents simulation experiment results regarding properties of linguistic pattern based simulated annealing used for solving the facilities layout problems in logistics. In the article, we investigate four different arrangements (02 × 18, 03 × 12, 04 × 09, and 06 × 06) comprising of 36 items. The examined layouts also differ in the links matrix density (20%, 40%, and 60%) and in defining distance between objects' pairs for the distance membership function (absolute and relative). We formally examine how these factors influence corrected mean truth values and average classical goal function values based on Manhattan distance metric. The results generally revealed a significant influence of all of the studied effects on the analyzed dependent variables. Some of the findings, however, were surprising and confirmed previous outcomes showing that the linguistic pattern approach is not a simple extension of the classic simulated annealing.

Keywords: facilities layout, optimization, linguistic variables, logistics, fuzzy sets.

1 Introduction

The main purpose of optimizing the placement of linked objects is to minimize the cost of their interoperability. In manufacturing engineering, one seeks such locations of machines and equipment in the production space that minimize the total cost of material flow, parts transportation, etc. between the objects. In the area of ergonomic research, the facilities layout problem is reduced to the arrangement of workplace elements that minimizes the overall cost of biological work such as energy expenditure. In the domain of human-machine interaction, the optimization could be based on the arrangement of the interface elements to ensure its maximum usability.

Since the layout optimization task is a combinatorial problem belonging to the NP-hard class [11], it is not possible to find optimal solutions within a reasonable time for large problems. Hence, the main research trend for years has been focused on developing heuristic algorithms leading to finding acceptable suboptimal solutions. Extensive reviews and classifications of such algorithms and optimization systems are presented, inter alia, in [8], [9], [13], [1] or [10].

An interesting research direction initiated, among others, by Scriabin and Vergin [12], focuses on the human ability to solve facilities layout problems. In general, studies conducted in this field has shown that intuition and experience allow experts to obtain very good solutions. Such results were a likely impulse for exploring the use of soft approaches in solving and modeling this type of problems. For instance, applications of fuzzy logic ([14], [15]) allow the use of experts' knowledge and experience in this respect. One of the possible approaches to modeling in such a manner was proposed by Grobelny [2], [3], [4], and [5]. The idea of linguistic patterns presented in these works enables the designers to take advantage of expert's intuition by expressing it in a form similar to natural language. The implementation of this concept in the simulated annealing algorithm [7] was proposed by Grobelny and Michalski in [6].

Although the overall nature of the approach and examples provided in this work are encouraging, the practical possibilities and limitations of the algorithm are not sufficiently examined and require further, systematic investigation. This paper presents the results of experimental studies on the influence of linguistic model describing the distance criterion and the links matrix density on the quality of solutions obtained by the proposed approach for a number of possible arrangements of 36 objects.

The following sections briefly discuss the idea of the linguistic pattern based approach, present the design of the experiment, the results along with discussion, and brief conclusions.

2 Overview of simulated annealing based on linguistic patterns

The implementation of the linguistic patterns concept in the simulated annealing algorithm proposed by Grobelny and Michalski [7] implies the substitution of the classical goal function formula with the following pattern (1).

$$\boldsymbol{If}\ Link\_between\_objects_{(ij)}\ is\ STRONG\ \boldsymbol{then}\ Distance\_between\_objects_{(ij)}\ is\ SMALL \quad (1)$$

When pattern (1) is true for all pairs of objects, we may assume that the solution is good. As it was demonstrated in a number of articles ([2], [3], [4], [5], [6]), it is possible to assess the truth level of formula (1) by means of the Łukasiewicz's implication formula (2):

$$Truth\_of(1) = minimum\{1, 1 - Truth\_of(l) + Truth\_of(r)\}, \quad (2)$$

where $Truth\_of(l)$ and $Truth\_of(r)$ denote truth values of left and right sides of linguistic pattern (1).

The overall idea of the simulated algorithm does not change, but the objects pairs changes are performed in each step based on the degree of mean truth for the solution.

3 Simulation experiment

3.1 Method

Experimental design. The experiment examines logistics problems comprising of 36 objects in various arrangements. The main three layouts include structures with two rows and 18 columns (02 × 18), three rows and 12 columns (03 × 12), as well as four rows and nine columns (04 × 09). For comparison purposes we also tested a typical square grid layout with 6 rows and columns (06 × 06). The layouts are schematically demonstrated in Fig. 1.

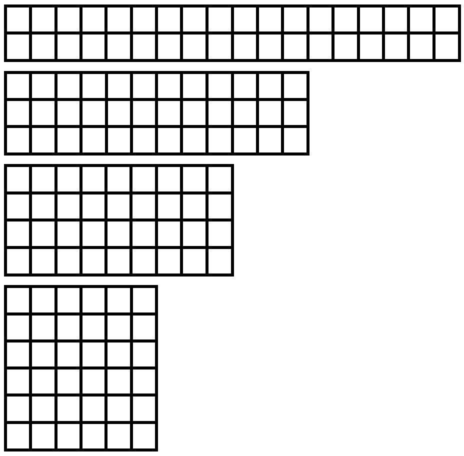

Fig. 1. Layout arrangements examined in this study. From the top: 02 × 18, 03 × 12, 04 × 09, and 06 × 06 layouts.

Relationships between objects were randomly generated from the 1 – 5 range. Their density was controlled and constituted the second independent variable specified on three levels 20%, 40%, and 60% of all possible links. The mean relationship value amounted respectively 2.90 (Standard Deviation, SD = 1.30; Flow Dominance, FD = 2.2), 2.96 (SD = 1.69, FD = 1.4), 3.19 (SD = 1.91, FD = 0.998).

The third factor investigated in this study was related with the way experts may treat the distance between objects. We tested two options: the relative and the absolute one. In the former approach, the membership function of the *SMALL* distance linguistic variable is linear starting from 1 for objects being next to each other and 0 when they were maximally far away. Such a case is symbolically illustrated in Fig. 5.

Naturally, the maximal distance varied along with the examined arrangements. Assuming the same, unitary distances between grid cells, the maximal Manhattan distance equals 10 for layout 06 × 06, 11 for 04 × 09, 13 for 03 × 12, and 18 for arrangement 02 × 18. In the absolute case, the distance memberships function is also linear and starts from 1 for adjacent items however, the zero value is assigned for all distances equal or bigger than 10. Thus, the function does not take into account differences in maximal possible gap between investigated arrangements. Graphical illustration of this idea is provided in Figs. 2 – 5.

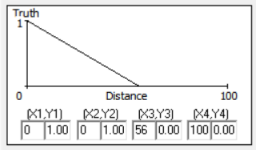

Fig. 2. Schematic presentation of the absolute distance membership function for 02 × 18.

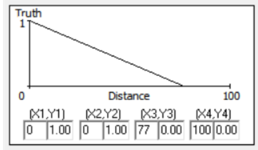

Fig. 3. Schematic presentation of the absolute distance membership function for 03 × 12.

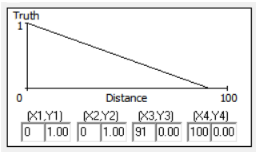

Fig. 4. Schematic presentation of the absolute distance membership function for 04 × 09.

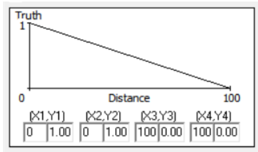

Fig. 5. Schematic presentation of the absolute and relative distance membership function for layout 06 × 06.

A full factorial, within subjects design gives 24 different experimental conditions, that is, arrangement {02 × 18, 03 × 12, 04 × 09, 06 × 06} × links density {20%, 40%, 60%} × distance function type {absolute, relative}. Since for layout 06 × 06 absolute and relative distance functions are exactly the same, the simulations finally included 21 variants.

The first dependent variable called corrected mean truth is based on goal function (3), which is related to the maximal possible mean truth value for a given arrangement (4) computed according to the theorem from [3].

$$mean\ truth = maximize\left(\frac{\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\left(Truth\_of(1)_{ij}\right)}{\frac{n^2-n}{2}}\right), \tag{3}$$

$$corrected\ mean\ truth = \frac{mean\ truth}{maximal\ mean\ truth} \cdot 100, \tag{4}$$

The second dependent measure included a typical goal function (5) based on Manhattan distance (6) that was computed for the best layouts obtained by maximizing mean truth (3).

$$\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\left(D_{p(i)p(j)} \cdot L_{ij}\right), \tag{5}$$

where $D_{p(i)p(j)}$ is the standard Manhattan distance metric calculated by the formula:

$$D_{p(i)p(j)} = \sum_{k=1}^{N}\left|x_{p(i)k} - x_{p(j)k}\right|, \tag{6}$$

with N denoting the number of dimensions – here equal two, *and* L_{ij} specifying the link strength between two objects.

Procedure. For each of 21 problems defined by the experimental design, the linguistic pattern based simulated annealing algorithm was repeated 100 times. Every time, before applying the optimization algorithm, objects were randomly assigned on the grid. Taking advantage of the experimental results presented in [6], in all present study simulations, the epoch length multiplier was set at 20, the probability of accepting worse solutions equaled .8, while the cooling scheme .99.

3.2 Results and discussion

Descriptive statistics. Basic descriptive statistics regarding all examined experimental conditions both for mean truth values and classical goal function values are put together in Table 1.

Table 1. Basic descriptive statistics for all experimental conditions and two dependent variables analyzed in the current study.

No.	Links density	Layout	Distance function type	Corrected mean truth			Manhattan based goal function		
				Max	Mean	MSE	Min	Mean	*MSE
1.	20%	02×18	Absolute	91.9	91.7	.0175	1387	1436	2.25
2.			Relative	98.6	98.5	.0095	1503	1532	1.94
3.		03×12	Absolute	96.4	96.2	.0111	1076	1123	2.08
4.			Relative	98.8	98.7	.0077	1113	1157	1.82
5.		04×09	Absolute	98.6	98.4	.0154	961	1005	1.71
6.			Relative	99.0	98.8	.0120	969	1007	1.70
7.		06×06		99.4	99.1	.0134	889	938	2.25
8.	40%	02×18	Absolute	85.0	84.9	.0096	3597	3646	2.74
9.			Relative	94.7	94.7	.0053	3655	3733	4.35
10.		03×12	Absolute	90.9	90.8	.0098	2722	2766	2.19
11.			Relative	95.0	94.9	.0069	2752	2817	2.87
12.		04×09	Absolute	94.1	94.0	.0111	2365	2428	2.69
13.			Relative	95.2	95.1	.0108	2383	2437	2.89
14.		06×06		95.6	95.5	.0088	2182	2257	2.26
15.	60%	02×18	Absolute	81.2	81.0	.0066	6499	6567	3.71
16.			Relative	92.0	91.9	.0029	6487	6558	4.97
17.		03×12	Absolute	87.9	87.8	.0088	4884	4940	2.36
18.			Relative	92.6	92.5	.0044	4871	4926	3.17
19.		04×09	Absolute	91.6	91.4	.0072	4235	4285.7	2.57
20.			Relative	93.0	92.9	.0060	4234	4286.4	3.36
21.		06×06		93.4	93.2	.0082	3941	4003	2.51

* MSE – Mean Standard Error

Analysis of variance. A series of analyses of variance were conducted to formally verify the influence of examined independent variables on two types of dependent measures. Summaries of essential results are provided in Figs. 6 – 20.

Corrected mean truths. The analyses' results of mean truth value depending on the investigated factors are graphically illustrated in Figs. 6 – 11. These figures show mean truth values for the studied arrangements from the perspective of two factors – the way the membership function is defined for linguistic variable $Distance\_between\_objects_{(ij)}$ is $SMALL$ and the link matrix density.

The obtained results were rather anticipated. The membership function for the relative approach treats bigger distances milder which makes it easier to obtain higher mean truth values for the same relationship matrix.

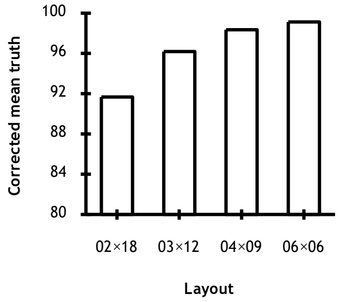

Fig. 6. Corrected mean truth depending on the layout. Links density: 20%, absolute distance membership function. $F(3, 396) = 53\,081$, $p < .0001$.

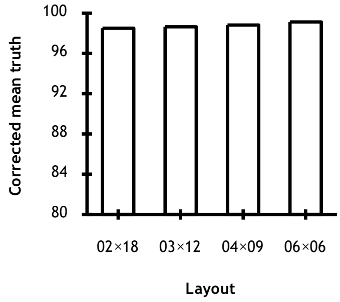

Fig. 7. Corrected mean truth depending on the layout. Links density: 20%, relative distance membership function. $F(3, 396) = 601$, $p < .0001$.

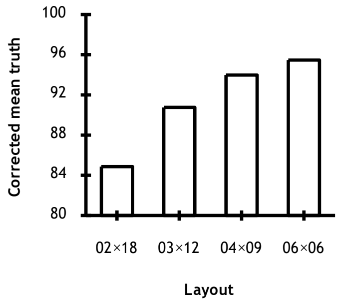

Fig. 8. Corrected mean truth depending on the layout. Links density: 40%, absolute distance membership function. $F(3, 396) = 226\,987$, $p < .0001$.

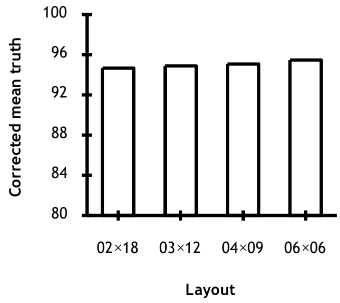

Fig. 9. Corrected mean truth depending on the layout. Links density: 40%, relative distance membership function. $F(3, 396) = 1700$, $p < .0001$.

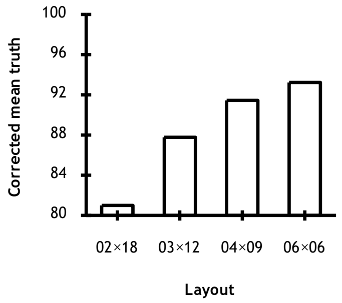

Fig. 10. Corrected mean truth depending on the layout. Links density: 60%, absolute distance membership function. $F(3, 396) = 488\,195$, $p < .0001$.

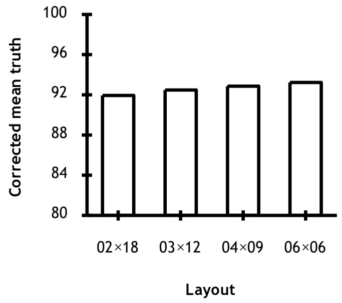

Fig. 11. Corrected mean truth depending on the layout. Links density: 60%, relative distance membership function. $F(3, 396) = 9377$, $p < .0001$.

It is also understandable that the value of average truth increases along with the bigger and bigger concentration degree of the layout arrangement. For more concentrated layouts, the area of possible solutions is characterized by smaller, on average, distances and the bigger number of available pairs of locations with the same distances. This results in greater freedom for the simulated annealing algorithm in making changes in objects locations by pairs.

The relative decrease in mean truth values along with the higher matrix densities was also predictable. In general, facilities layout problem tasks are more difficult to solve for dense relationships between objects. It is worth pointing out that all differences in means presented in Figs. 6 – 11 are statistically significant.

Goal function based on Manhattan distance. The outcomes for the classical objective function (5) are demonstrated in Figs. 12 – 20. . It seems that the most important relations were observed for definitions of membership functions in the absolute and relative conventions. One may be surprised while comparing these results with those presented in Figs. 6 – 11. Function values (5) express the overall cost and are to be minimized, so they should be interpreted inversely than corrected mean truths that supposed to be maximized. For small and medium densities, classic goal function mean values are better for distances treated in an absolute manner.

Comparison with the results from Figs. 6 – 11 clearly indicates that, in terms of mean truth of pattern (1), better solutions were obtained for the membership functions representing relative distances. This dependence disappears as the relationship density increases. For a density of 60% it becomes even, slightly but statistically significantly, reversed (Table 2 and Fig. 20). Such a reverse relation is strongly influenced by the shape of the available area modeled by the modular grid structure.

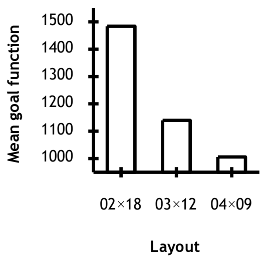

Fig. 12. Mean goal function depending on the layout. Links density: 20%. $F(2, 594) = 32\,660$, $p < .0001$.

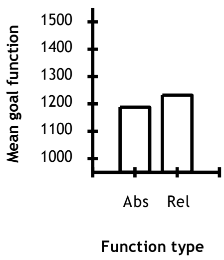

Fig. 13. Mean goal function depending on the function type. Links density: 20%. $F(1, 594) = 783$, $p < .0001$.

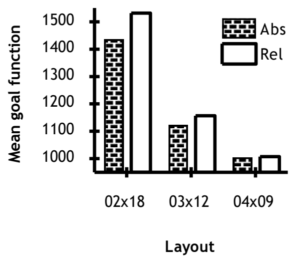

Fig. 14. Mean goal function depending on the layout×function type. Links density: 20%. $F(2, 594) = 302$, $p < .0001$.

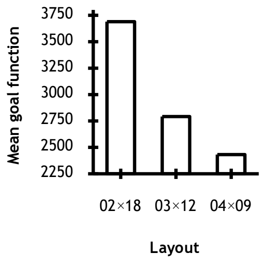

Fig. 15. Mean goal function depending on the layout. Links density: 40%. $F(2, 594) = 91\,506$, $p < .0001$.

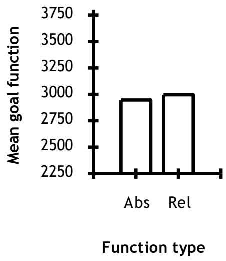

Fig. 16. Mean goal function depending on the function type. Links density: 40%. $F(1, 594) = 399$, $p < .0001$.

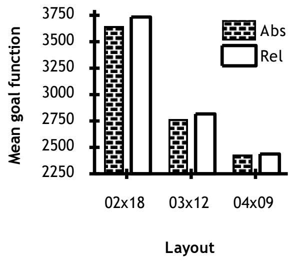

Fig. 17. Mean goal function depending on the layout×function type. Links density: 40%. $F(2, 594) = 83$, $p < .0001$.

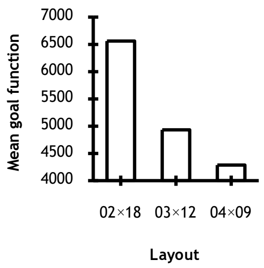

Fig. 18. Mean goal function depending on the layout. Links density: 60%. $F(2, 594) = 229\,500$, $p < .0001$.

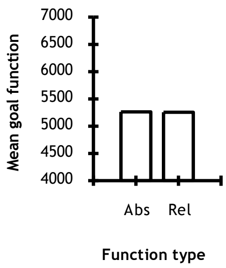

Fig. 19. Mean goal function depending on the function type. Links density: 60%. $F(1, 594) = 7.1$, $p = .0078$.

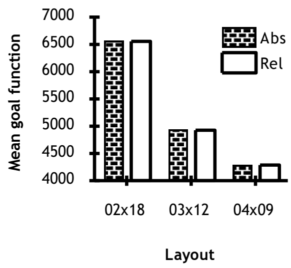

Fig. 20. Mean goal function depending on the layout×function type. Links density: 60%. $F(2, 594) = 2.3$, $p = .096$.

Table 2. Manhattan based mean goal function differences between absolute and relative membership function types

	Links density								
	20%			40%			60%		
	02×18	03×12	04×09	02×18	03×12	04×09	02×18	03×12	04×09
Absolute vs Relative LSD probability	< .001*	< .001*	.34	< .001*	< .001*	.028*	.056**	.0045*	.88

* $\alpha = .05$, ** $\alpha = .1$

4 Conclusion

Grobelny and Michalski in [6] showed that the simulated annealing algorithm based on linguistic patterns is not a simple extension or generalization of the traditional approach where the function (5) is minimized. The results obtained in the current experiments empower the specifics of the approach and suggest that subtle analysis of its properties may lead to interesting outcomes.

The presented empirical data both in the present study as well as in the work [6] reveal that the behavior of the investigated algorithm with linguistic patterns is determined by many factors and interactions between them. In light of this, further research on the properties of the approach is justifiable and necessary.

Usually, in practical applications of the proposed approach, the actual relationships and distances are not known and are estimated in linguistic terms by an expert. Despite that, findings presented in the current study are an important indication for rational use of linguistic pattern based simulated annealing.

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